Experimental Investigation of a Nonlinear Elastic Suspension

T. E. SHOUP*
Rutgers University, New Brunswick, N.J.

THE design of linear isolation mounts is usually accomplished for a best compromise between softness for steady-state vibration isolation and stiffness for transient or shock isolation. It has been shown[1–3] that nonlinear elastic suspensions that stiffen as they deflect combine the advantages of soft and stiff systems to provide improved designs for isolation of both transient and steady-state inputs. In a previous Note[4] a suspension system known as an "elastica" suspension was presented and was theoretically analyzed for static and dynamic behavior. Because of its simple construction and nonlinear behavior, this device is well suited for aerospace and transportation applications. The purpose of this Note is to provide an experimental verification of the theoretical behavior predicted in the previous Note.

Elastica Suspension System

The elastica suspension spring is constructed from a pair of thin flexible strips clamped in a semicircular shape in the undeflected position. A suspension system constructed from two such elastica spring pairs in parallel is shown in Fig. 1. For this device the length of the flexible strips is represented by L, the flexural rigidity of the strips is represented by EI, and the mass of the suspended platform is represented by m. As the outer frame of this

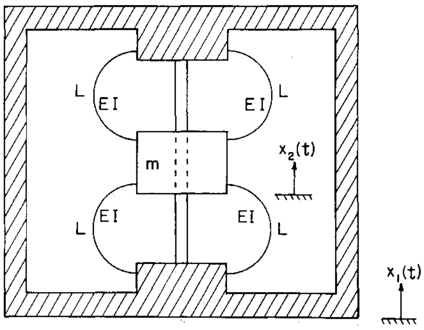

Fig. 1 "Elastica" suspension system consisting of two spring pairs in parallel.

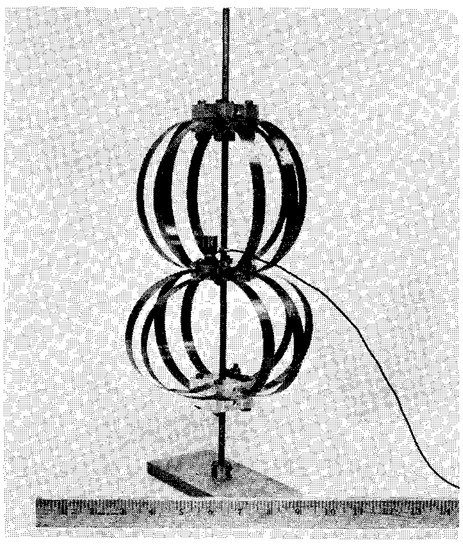

Fig. 2 Experimental model of the "elastica" suspension system.

Received November 23, 1971. The author gratefully acknowledges the financial assistance given by the Rutgers University Research Council in support of this investigation.

* Assistant Professor of Mechanical and Aerospace Engineering.

device is moved with displacement $x_1(t)$, the platform moves with displacement $x_2(t)$.

The experimental device constructed to verify the theoretical curves is shown in Fig. 2. The eight spring pairs used in the construction of this device consist of sixteen pieces of 0.005 in. × $\frac{1}{2}$ in. tempered steel feeler stock with $L = 8.0$ in. In order to minimize damping due to coulomb friction, the center platform contains a $\frac{1}{4}$ in. ball bushing that slides on a case hardened and ground shaft. The combined weight of the platform and piezoelectric accelerometer displacement transducer gives rise to a natural frequency for small deflections of $\omega_n = 4.78$ Hz. For this device deflections of $X/L = 0.2$ give rise to a maximum bending stress of 87,000 psi.

Experimental Behavior

The static load vs deflection data points have been plotted on Fig. 3 in nondimensional form to facilitate comparison with the theoretical curve developed in the previous Note.[4] Data points for this figure were obtained using a set of standard weights and a vernier caliper. The measurements were made in the vertical direction with respect to gravity, and the weight of the center platform was included as part of the applied load.

The dynamic response of this system was tested with the system in a horizontal orientation in order to eliminate the effect of gravity. The system was subjected to an input of $x_1(t) = X_1 \sin \omega t$, and the observed output was closely sinusoidal of the form $x_2(t) = X_2 \sin(\omega t + \phi)$. The frequency response data points have been plotted on Fig. 4 in nondimensional form to facilitate comparison with the theoretical curves developed in the previous Note.[4] The displacement output $x_2(t)$ for these tests was measured using a piezoelectric accelerometer, double integrator, and a lightbeam oscillographic recorder. Input displacement $x_1(t)$ for the $X_1/L = 0.01$ and 0.05 curves was obtained using an electromagnetic shaker and input for the $X_1/L = 0.1$ curve was obtained using a mechanical linkage shaker. The free response data points ($X_1/L = 0$) were obtained by plucking the platform and observing the variation of frequency of vibration with respect to amplitude. All experimental measurements presented in Figs. 3

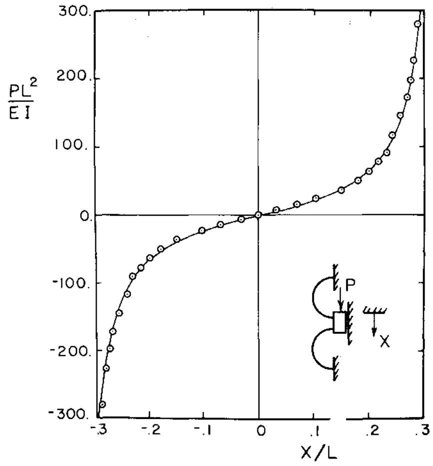

Fig. 3 Nondimensional load vs deflection curve for the "elastica" suspension spring pair.

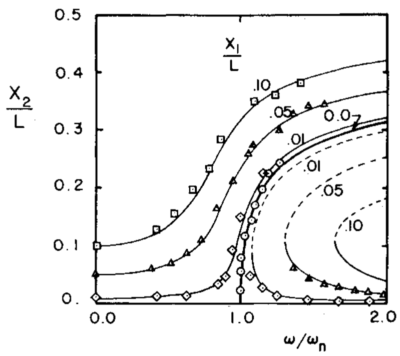

Fig. 4 Frequency response of the "elastica" suspension system.

and 4 have an uncertainty of less than 5%, and the correlation between theoretical and experimental results is quite good.

The characteristic "jump" behavior associated with nonlinear systems was observed in these tests. Branches of X_1/L to the left of the free response curve in Fig. 4 correspond to motion that is in phase with the input, while branches to the right of the free response curve exhibit motion that is 180° out of phase with the input. As the vibration frequency ω/ω_n is increased from zero the X_2/L values will follow the upper X_1/L curve until a sudden "jump" to the lower X_1/L curve occurs. As the vibration frequency ω/ω_n is decreased from a high value, the motion X_2/L is observed to jump from the lower X_1/L to the upper X_1/L curve.

One of the advantages of the "elastica" suspension spring is that it can easily be modified to provide a different behavior than presented in Figs. 3 and 4. For example, if the fixed ends of the upper and lower "elastica" springs are moved apart (so as to violate the condition of initial semicircular shape) the load vs deflection curve becomes stiffer than predicted by Fig. 3. Conversely, if one attaches the fixed ends closer together, the load vs deflection curve will become softer than predicted by Fig. 3.

References

[1] Efstathiades, G. J. and Williams, C. J. H., "Vibration Isolation Using Non-Linear Springs," *International Journal of Mechanical Science*, Vol. 9, 1967, pp. 27–44.

[2] Tobias, S. A., "Design of Small Isolator Units for the Suppression of Low-Frequency Vibration," *Journal of Mechanical Engineering Science*, Vol. 1, No. 3, 1959, pp. 280–292.

[3] Young, D., Barton, M. V., and Fung, Y. C., "Shock Spectra for Nonlinear Spring-Mass Systems and Their Applications to Design," *AIAA Journal*, Vol. 1, No. 7, July 1963, pp. 1597–1602.

[4] Shoup, T. E., "Shock and Vibration Isolation Using a Nonlinear Elastic Suspension," *AIAA Journal*, Vol. 9, No. 8, Aug. 1971, pp. 1643–1645.